

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

I. Pre-Lab Introduction

Please complete the following pages before the lab by filling in the requested items.

A. Atomic notation:

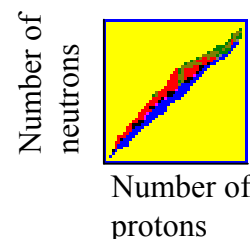
Atoms are composed of a nucleus containing neutrons and protons surrounded by a cloud of electrons. A given chemical element contains a fixed number of protons in the nucleus but there may be different numbers of neutrons. Atoms of a given element that have different numbers of neutrons are called *isotopes* of that element. A neutral atom has the same number of electrons as protons. The notation for an atom is given by the symbols ${}^A_Z X$ where X stands for the chemical symbol of the element, Z the number of protons or the *atomic number* and A is called the *atomic mass number* and it is equal to the sum of the number of protons plus the number of neutrons. For example, carbon-14 is written as ${}^{14}_6 C$ since this neutral isotope of carbon has 6 protons, 8 neutrons, and 6 electrons.

1. a. Write the symbolic notation for a neutral radium atom that has 226 nucleons:
b. How many protons does this atom have? _____
c. How many neutrons does this atom have? _____
d. How many electrons does this atom have? _____



It is possible in principle to combine any number of neutrons with any number of protons. However, that does not mean that any combination of nucleons will form a stable nucleus. A graph of the number of neutrons versus the number of protons in stable nuclei forms a “stability curve” also called a “table of isotopes”. If by some means, you manage to create a nucleus with a relative neutron and proton number far from this “stability curve”, it would simply fly apart. However, if you create a nucleus with only slightly too many or too few neutrons, the nucleus merely decays to a more stable arrangement by the emission of some particle.

Radioactivity is the emission of particles by unstable nuclei.



B. Radioactive Decay:

A basic concept of radioactive decay is that the probability of decay for each type of radioactive nuclide is constant and is characterized by a number called the decay constant a . In other words, there are a predictable number of decays per second even if it is not possible to predict which nuclei in the sample will decay. If you have a sample of N radioactive nuclei there will be some number (ΔN) that will decay during a measured amount of time (Δt). The rate that these nuclei decay depends on how many nuclei there are since the process of any given atom decaying is a random event.

$$\frac{\Delta N}{\Delta t} = -aN \quad (1)$$

The symbols $\Delta N/\Delta t$ stand for the rate of change of N with time t .

2. (Circle one) The negative sign in the above equation indicates that the number of nuclei are (**increasing** / **remaining constant** / **decreasing**).

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

The question of interest is: How many radioactive nuclei N_f are left after some time t has passed? The answer to that question is found by solving equation (1) above by using a method from calculus called integration.¹ The solution is

$$N_f = N_i e^{-at} \quad (2)$$

where e is the base of natural logarithms ($e = 2.718\dots$). Equation (2) states that the number of radioactive nuclei remaining N_f after some elapsed time t decreases exponentially from an initial number N_i of radioactive atoms.

3. What is the value of N_f when $t = 0$? _____
4. What is the value of N_f when $t = \infty$ (becomes very large)? _____

The initial number of radioactive nuclei is designated as N_i . If this occurs at time $t = 0$, the initial value, N_i , is often labeled N_0 .

For laboratory measurements the counting rate, R , is the number of decay particles that pass through a detector per unit time is directly proportional to the number of remaining radioactive nuclei N_f , and the initial rate R_i , is directly proportional to initial number of nuclei N_i .

5. Determine the equation relating the counting rate R to the initial rate R_i , the decay constant a , and the time t .

$$R = \underline{\hspace{2cm}}$$

C. Half-life:

An important concept associated with the radioactive decay process is the *half-life*. The time interval for a radioactive sample to go from N_i nuclei or a rate of R_i to one-half that value $N_i/2$ or $R_i/2$ is defined as the half-life, $t_{1/2}$.

6. Determine the equation for the half-life in terms and a . Set R/R_i equal to $1/2$, take the natural log, and solve for $t_{1/2}$: (fill in the gray squares with the appropriate values)

$$\frac{R}{R_i} = \frac{1}{2} = e^{-at_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = \ln(e^{-at_{1/2}})$$

$$-\ln(2) = -at_{1/2}$$

$$t_{1/2} = \frac{\square}{\square}$$

¹ Derivation of equation (2) can be found at the end of this laboratory activity.

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

7. Now it is possible to determine a from the decay rate measurement. Transform the equation $R = R_i e^{-at}$ by taking the natural logarithm of both sides and **solve for a** :

$$\ln(R) = \ln(R_i e^{-at})$$

$$\ln(R) = \ln(R_i) + \ln(e^{-at})$$

$$\ln(R) = \ln(R_i) - at$$

$$a = \underline{\hspace{2cm}}$$

Since R_i is a constant, this equation predicts that the graph of $\ln(R)$ versus t will be a straight line with slope a . An a with a negative value is a downward sloped line.

II. Experiment

Objective

In this experiment a sample will be made radioactive by placing it in a neutron generator where it will be bombarded with neutrons. After the sample has been “activated”, the sample’s rate of decay (R) will be measured and recorded at regular time intervals. From these data, you will be able to determine the sample’s half-life and the element isotope activated.

A. Apparatus:

This experiment uses a Geiger tube and counting unit, computer data acquisition program and a neutron-activated sample. The Website shows a video recording of several experiments that were performed. A computer data acquisition program was used to record the number of counts the Geiger tube and counting unit received. The data were saved as text and Excel tables and can be found on the Website.

Your instructor will either show you one of the videos or have you select an object to see. Watch the background measurement, the experiment, and a data file of the results from that experiment.

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

B. Background Count rate example seen in the Video.

In order to properly count the decay rate of a sample, a background measurement must first be taken. The background measurement records the decay counts from naturally occurring sources. Watch the "Background Measurement" video to see how a background measurement is performed.

1. What was the Background Count rate seen in the video?

Number of Background Counts during 10 minutes	Background Count Rate (c/m)

NOTE: For your experiment, your data set may indicate a different background count rate. This example is intended to show how to determine the background count rate in counts per minute.

C. Radioactive Decay of an Activated Sample

Choose one sample that you will determine the half-life of. Samples to investigate can be found on the Website.

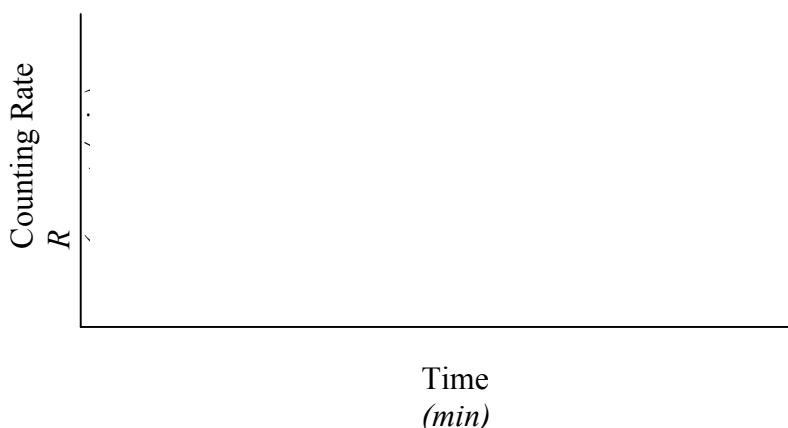
2. I will investigate the half-life of _____

Watch the video showing the neutron activation and radioactive decay for your sample. While watching the video, answer the following questions:

3. Describe how the activity (represented by the Geiger counter response) in the sample seen in the video during the following times compares to the background rate seen earlier. Use a scale of 0 (representing no radioactive decays) to 10 (a very large number of decays per minute occurring):
 - a. Before activation: _____
 - b. At the beginning of counting after activation: _____
 - c. At a time in the middle of the experiment: _____
 - d. At the end of the experiment: _____
4. How long was the sample activated in the Neutron source? _____
5. How long was the count time (how long did each measurement last)? _____
6. How many count runs occurred? _____

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

7. Based on what you saw in the video, make a qualitative graph of what you think the counting rate R (decays per minute) looks like as time progresses.



8. (Circle one) As time progresses the number of **radioactive** atoms in the sample (**increases** / **decreases** / **remains the same number**).
9. (Circle one) As time progresses the number of **all** atoms in the sample (**increases** / **decreases** / **remains the same number**).

III. Data Analysis

A. Analysis Overview:

To determine the half-life of your sample, you will need to graph your data. In order to properly graph your data, you will need to plot the number of counts per minute (Rate) less any background counts on the y -axis vs. the time when the data was taken on the x -axis. After plotting your data in this manner, you will also plot the natural logarithm of the count rate, $\ln(R)$, vs. time. The data in this second graph should show a line. The slope of the line formed by the data is the decay constant a as determined in section I. C. 7.

If you have extra time, after determining the decay constant, and associated half-life, you can fit your data to the radioactive decay equations to perform some basic data modeling.

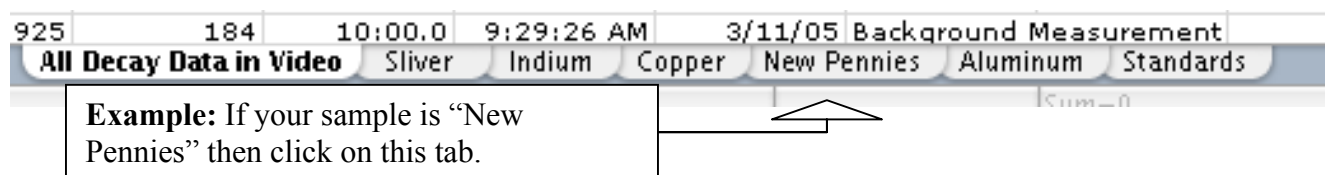
In short, you will:

1. Determine the "Average Counts per Minute" and complete the values for that column. Another way to think about this, is that this column represents how many counts there should be if counting occurred for a minute.
2. Determine the corrected count rate R_c by subtracting the background value from the "Average Counts per Minute" column.
3. Calculate the natural log of the corrected count rate: $\ln(R_c)$ and determine the slope of the graph to find a and the sample's half-life $t_{1/2}$.

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

B. Find your Data for Analysis

Open the Excel file: “Student Decay_Data.xls”. In the file, click on the tab at the bottom of the Excel page relating to your chosen sample:



Example: If your sample is “New Pennies” then click on this tab.

The data shown in the tabbed page are the data you saw being acquired by the computer in the video. You will analyze this data to determine the half-life of your chosen element.

C. Time Data Analysis (Column K)

As you saw in the video, the data counts began at 0 at the start of the count run, and ended with the recorded value at the end of the count run. The count value recorded in column C of the table thus represents the average count rate over a counting period. The relevant question to answer is: How do you determine what values of time to use?

Since the counts change over time, the counts (representing the decays) collected during a given time period is an *average* value for that time period. You need to determine the values to place in the cells for the “Time” column that occur at the midway point for each data collection run. For **example**, if the count process occurred in 30-second intervals, the time of the average counts for the first run (0 – 30 seconds) is 15 seconds and 15 is entered into table cell K8. The second run (30 – 60 seconds since the start of the entire counting portion of the experiment) has the time of the second run added the 15 second average found from the first to yield 45 s (since $15 + 30 = 45$) and 45 is entered into cell K9. The third run (60 – 90 s since the start of the experiment), the time taken to complete the third run is added to that found for the second yielding a value of 75 s. (since $45 + 30 = 75$) with 75 entered into cell K10, etc.

1. Determine all the values for the “Time” (column K) for your sample. Pay attention to the time intervals and if any delays occur between data collection times as those will make your times differ from the times in the above example. Time “0” should be entered in cell K7.

D. Background Rate

Check the background measurement recorded for your sample and calculate the background count rate (counts per minute: CPM). The background measurement will be the first count run found on **row 7** of the table and will look similar to the **example** below.

Run#	Voltage	Counts	Time	Time Of Day	Date	Notes:
1	925	226	10:00.0	5:08:16 PM	3/10/05	Background Measurement

The total number of counts is in this cell.

The total time is in this cell.

1. Our sample’s background rate is: _____

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

E. Average Counts per Minute (Column L), correction for the Background (Column M), and $\ln(R)$ (Column N)

1. Complete the Average Counts per Minute for your element. (Column L) **Start your data in cell L8.** You can use an Excel formula to copy data from one cell to another. For **example**, **IF** your data count run time was **15 seconds** **THEN** the counts occurring in one minute (counts per minute) would be **four** times the recorded number. The Excel formula for the cell L8, **IN THIS EXAMPLE**, is (NOTE: your equation will be slightly different):

$$=C8 * 4$$

Pressing return will calculate the value for the cell. If you then put the cursor in the lower right hand corner of the cell it will change to a solid plus (+), and if you click and drag down the column, the formula, with the referenced cell automatically incremented, will be copied to the highlighted cells.

2. Correct your Counts Per Minute (c/m) data for the background value found in step **III. D. 1.** by subtracting the background count rate, and record as the corrected count rate R . (Column M, begin in cell M8).
3. Using the corrected count rate, record the values of the natural log of R : $\ln R$ for all your data. (Column N, begin in cell N8). If your count rate R is in cell M8, Excel can calculate this value with the cell equation of:

$$=\ln(M8)$$

F. Determination of the half-life from the R_c versus t graph:

Plot the corrected count rate R_c versus time as an X-Y (Scatter) chart without lines. Either hand-draw a SMOOTH curve that fits your data as well as possible or use the Chart → “Add Trendline” option that best fits your data.

Extend the graph line (best guess) back to $t = 0$ and find an estimate of R_0 , the count rate at $t = 0$. Enter this value in cell M7 and calculate the value for cell N7.

1. The value of R_c at $t = 0$ is called R_0 . On your graph, determine R_0 : _____ c/m
2. On your graph determine the value of $R_0 / 2$: _____ c/m
3. The time when $R_c = R_0 / 2$ is: _____ seconds.

This time when the count rate is half your initial value is the half-life for your sample; this is the time it takes for half of the initial radioactive atoms to decay.

For a more accurate value of the half-life of your sample, find the time for $\frac{1}{4}$ (two half-lives) of the initial count rate R_0 , or for $\frac{1}{8}$ (three half-lives) and use that time to calculate your half-life (dividing by 2 or 3 as necessary). Depending on your chosen sample, you may be able to see one, two, or three half-lives on your graph.

4. Our sample's R vs. t graph shows _____ half-lives. The sample's half-life is _____ sec.

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

G. Determination of the half-life from the $\ln(R_c)$ versus t graph:

Plot $\ln(R_c)$ versus time as an X-Y Scatter chart, draw a best-fit straight-line for your data, and determine the slope (rise/run) of this graph. To find the slope, either use two points far apart on the fit line or use the trendline equation feature by selecting Chart → “Add Trendline ...” then choose “Options” and check “Display equation on chart”.

1. Find two convenient and well-separated points on your graph. More accurate results are obtained when the two points you select are as far apart as possible. Label the points 1 and 2. Do not just use your data points because they may not lie exactly on the line.

The value of $\ln R_c$ at point 1: _____

The value of $\ln R_c$ at point 2: _____

The “rise” [$\Delta \ln(R_c)$] of the line between points 1 and 2: _____

The value of t at point 1: _____ sec

The value of t at point 2: _____ sec

The “run” [Δtime] of the line between points 1 and 2: _____ seconds

The slope, a , of the line = rise/run. $a = \frac{\Delta \ln(R_c)}{\Delta t}$

(include units, and keep at three significant digits): slope = _____

2. The decay constant for element _____ is $a =$ _____.

3. Compute the half-life from the appropriate equation in the pre-lab activity (Section I. C. 6). Show your work.

The value of the half-life from the $\ln R_c$ versus t graph is _____ sec.

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

4. Is this a similar value to that which you determined in part A? Calculate the % error between the two methods.

5. Which value do you think is more accurate? Why?

IV. Fitting your Data to Theory

Now that you have determined the half-life of your sample, you can plot the theoretical decay curve on the same graph as your data.

1. In Column O, calculate the theoretical decay curve given your initial decay rate R_0 and your decay constant a using the equation: $R = R_0e^{-at}$. In cell O8, enter the following Excel equation (replacing the brackets with the values determined in the experiment):

$$= [\text{your value for } R_0] * \exp(-[\text{your value for } a] * L8)$$

2. Add the results as a second data series as a “theoretical” curve (line, no data markers) to your R vs. t graph and include your graph with your lab report.

3. In Column P, calculate the log of the theoretical decay curve given your initial decay rate R_0 and your decay constant a using the equation: $\ln(R) = \ln(R_0e^{-at})$. Simply take the natural logarithm of the values you calculated in step 1. In cell P8, enter the following Excel equation:

$$= \ln(O8)$$

4. Add the results as a second data series as a “theoretical” curve (line, no data markers) to your $\ln(R)$ vs. t graph and also include this second graph with your lab report.

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

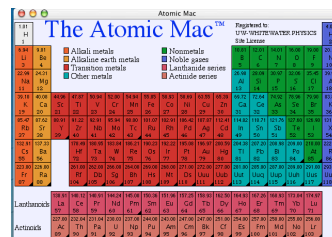
V. Independent Check

Determine the isotopes of the element you are investigating. You can use one of the following Web sites (or similar) located at

<http://www.nndc.bnl.gov/nudat2/>

<http://ie.lbl.gov/toi/perchart.htm>

<http://www.webelements.com/>



or from a printed version such as the CRC Handbook of Chemistry and Physics. Retrieve information on your element and its isotopes.

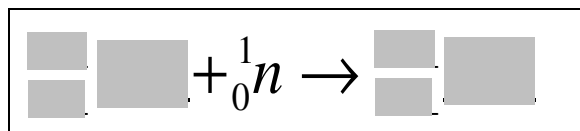
1. For your sample element, which isotope has a half-life decay time close to your experimental value?

The sample was prepared by neutron activation: a neutron is added to a stable nucleus which then undergoes decay since the extra neutron makes the nucleus unstable.

2. Which is the naturally occurring (stable) isotope in your sample that is irradiated?
3. Why do you think so?

4. Complete the following neutron activation expression for your sample (fill in the gray squares). Choose the isotope that you believe is responsible for the decays that were observed.

Activation Process



5. In beta decay, a neutron in the nucleus decays into a proton, electron, and an anti-neutrino. Complete the decay expression for this type of decay with the appropriate isotopes from part 3. Also, indicate accepted half-life from your research.

Decay Process	Decay Mode	Half-life
$\begin{array}{ c } \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \square \\ \hline \end{array} \Rightarrow \begin{array}{ c } \hline \square \\ \hline \square \\ \hline \end{array} \begin{array}{ c } \hline \square \\ \hline \square \\ \hline \end{array} + {}_{-1}^0e + \bar{\nu}$	Beta	

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

6. Compare your half-life values with Accepted value. Determine the percent error of your experimental values for the half-life to the accepted value.

VI. Conclusion.

Write a concluding statement about what you learned in this laboratory, what you discovered, which isotope produced the decay, the experimental half-life you determined, and the error.

THE HALF-LIFE OF A RADIOACTIVE ELEMENT

Appendix

Derivation of equation (2):

If you have a sample of N radioactive nuclei there will be some number (ΔN) that will decay during a measured amount of time (Δt). The rate that these nuclei decay depends on how many nuclei there are times a decay constant:

$$\frac{\Delta N}{\Delta t} = -aN \quad (1)$$

Taking the limit of $t \rightarrow 0$, yields a differential which can be integrated.

$$\lim_{t \rightarrow 0} \frac{\Delta N}{\Delta t} = \frac{dN}{dt} = -aN, \text{ so}$$

$$\frac{1}{N} dN = -a dt, \text{ integrating from } 0 \text{ to } t' \text{ yields:}$$

$$\int_{N_i}^{N_f} \frac{1}{N} dN = -a \int_0^{t'} dt$$

$$\ln(N_f) - \ln(N_i) = -a(t' - 0)$$

$$\ln\left(\frac{N_f}{N_i}\right) = -at', \text{ raising both sides by the power of } e \text{ (base of natural logarithms = 2.718 ...)}$$

$$\frac{N_f}{N_i} = e^{-at'}$$

$$N_f = N_i e^{-at'}$$

Thus, re-labeling t' by t yields the result:

$$N_f = N_i e^{-at} \quad (2)$$