

Note for the Decay Graph Equation:

The equation $N = N_0 e^{-(\ln 2)(\text{turn \#})}$ used for the theoretical plot of the “Graph of the Decay for 100 Coins” comes from the exponential law of decay, where the remaining number of coins N , is equal to the initial number N_0 times the decay value of $e^{-\lambda t}$, with λ being the decay constant and t the time :

$$N = N_0 e^{-\lambda t}.$$

When half of the coins have “decayed,” $N = N_0/2$. Since each coin has a 50-50 probability of being heads-up or down, each toss is equivalent to one half-life time ($t_{1/2}$). Thus the above equation becomes

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}.$$

Now, the initial number of coins, N_0 can cancel and the natural log of both sides of the equation is taken to give the relation

$$\ln\left(\frac{1}{2}\right) = \ln(e^{-\lambda t_{1/2}}).$$

But since $\ln(1/2) = -\ln(2)$ and $\ln(e^x) = x$ the equation becomes

$-\ln(2) = -\lambda t_{1/2}$, or rearranging terms: $\lambda = \frac{\ln(2)}{t_{1/2}}$. Replacing this relationship for the

decay value in the initial equation yields

$$N = N_0 e^{-\frac{\ln(2)}{t_{1/2}} t} = N_0 e^{-(\ln(2))\left(\frac{t}{t_{1/2}}\right)}$$

However, the fraction $\frac{t}{t_{1/2}}$ which is the total time divided by the half-life time is just the number of cup tosses (or turns) given to the pennies since each toss represents one half life. This gives the remaining number of coins as:

$$N = N_0 e^{-(\ln 2)(\text{turn \#})}.$$

Note for the Growth Graph Equation:

The equation

$$N = N_0 e^{(\text{turn \#}) \ln(3/2)}$$

used for the theoretical plot of the “Graph of the Growth of Coins” comes from the exponential law,

$$N = N_0 e^{\lambda t}$$

where the number of coins N , is equal to the initial number N_0 times the exponential growth factor $e^{\lambda t}$, with λ being the growth constant and t the time. This is directly related to the algebraic form of the equation of

$$N = N_0 (1 + 0.5)^{t/p}$$

Where factor of 0.5 is the percent increase (0.5 chance of being heads on the coin), and p is the time period of time for the increase. Thus

$$N_0 e^{\lambda t} = N_0 (1 + 0.5)^{t/p}$$

can be reduced by dividing by N_0 and taking the natural log:

$$\ln(e^{\lambda t}) = \ln((1 + 0.5)^{t/p})$$

$$\lambda t = \frac{t}{p} \ln(1 + 0.5)$$

$$\lambda = \frac{1}{p} \ln\left(\frac{3}{2}\right)$$

Putting this value of λ in for the growth constant back into the original equation yields:

$$N = N_0 e^{\frac{t}{p} \ln(3/2)}$$

But the ratio of the elapsed time t over the time period for the increase p is the turn #. So the result is:

$$N = N_0 e^{(\text{turn \#}) \ln(3/2)}$$

Note: The graph that students get will be determined somewhat on if they are “lucky” getting heads when they have a single coin. If they are “unlucky” their graph will be shifted to the right from the theory set as it will take them a few more tosses to get their numbers to increase.